# ON THE MATHEMATICAL BASES OF EINSTEIN'S STR

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**Abstract**

The concept of the Dynamic and Kinematic reference systems, according to Newton [1], [2], enables to co-ordinate the Einstein's axioms of the Special relativity theory with the mathematical contents of this theory. The first axiom of SRT is not a generalization of the Galileo-Newton relativity principle to the electrodynamics processes, as was assumed by Einstein, but mathematical requirement of covariance of equations of physics relative to the Lorentz transformations. The second axiom of SRT is not the statement on the constancy of the light velocity $c = const$ for inertial reference systems, as per Einstein's assumption, but mathematical requirement of invariance $c = inv$ of this velocity relative to the Lorentz transformations. As a result, SRT is based not on the Einstein's statement [3] "The following reflexions are based on the principle of relativity and on the principle of the constancy of the velocity of light", but on the statement "The following reflexions are based on the principle of covariance and on the principle of the invariance of the velocity of light". Ignoring this circumstance leads to the contradictions and paradoxes, being the objects of the critics of Einstein's theory from the moment of its appearance up to our days.

Key words: dynamic and kinematic reference systems; inertial reference systems; relativity principle; covariance and invariance of equations of motion.

## Definitions

According to Newton all movable reference systems are divided into two classes – dynamic and kinematic [1], [2].

If the considered system of material particles is moving simultaneously with the reference system $\Sigma$, which, in turn, is moving relative to the sphere of remote stars, then $\Sigma$ is called a dynamic reference system for the given process. If the considered system of material particles does not participate in the transport motion together with the reference system $\Sigma'$, then the latter is called kinematic reference system for the given process. It is worth to note the relativity of these concepts: the same reference system for one process can be dynamic, while for the others – the kinematic.

The dynamic reference systems, in turn, are subdivided into two classes - inertial and non-inertial ones. Just in dynamics a concept of inertial reference systems appears. Dynamic reference systems, which move translational, uniformly and rectilinearly concerning the sphere of remote stars, therefore, concerning each other as well, are called the inertial reference systems. In each of such dynamic inertial reference systems, according to the experimentally established dynamic relativity principle of Galileo-Newton, physical laws of not mechanics only, but also of the electrodynamics, are stated and written down equally to within notation of coordinates. Equations of motion of physical processes in dynamic inertial reference systems never include speed of their motion relative to the other reference systems. The dynamic reference systems, which move with acceleration with respect to the inertial dynamic reference systems, are called the non-inertial dynamic reference systems.

## Newton dynamics equations are Galileo-covariant for all kinematic reference systems

Let us write the fundamental equation of the material particle $M$ in the inertial reference system $Oxyz$ under some initial conditions

$$m\bar{a} = F(t, \bar{r}, \bar{\mathrm{v}}) \qquad (1)$$

According to the dynamic Galileo-Newton's relativity principle, in any other inertial reference system $O^*x^*y^*z^*$ the motion of another material particle $M^*$ with the same value of the mass $m$ is described by the same equation

$$m\bar{a}^* = F(t, \bar{r}^*, \bar{v}^*) \quad (2)$$

If initial conditions for $M^*$ in the reference system $O^*x^*y^*z^*$ are accurately the same as for $M$ in the reference system $Oxyz$, then we come to the following formulation of the Galileo-Newton's relativity principle: the identical mechanical processes, each of which occurs in their inertial reference systems, are observed and described identically with an accuracy to the designations of coordinates. This principle is a consequence of experimental fact. At the end the 19$^{th}$ Century the fulfillment of this principle was experimentally confirmed also for the electrodynamics processes. Definite difficulties with fulfillment of this principle arose only in optics. This is explained by the fact that the concept of inertial reference systems is not applicable to the photon. The photon, emitted by source, in contrast to the material particle, does not preserve the speed of motion of its source. Therefore the first Newton's law, law of inertia, is not applicable to the photon. Any reference system for the photon is kinematic.

In the kinematic reference systems there exists a mathematical principle, analogous to the experimental Galileo-Newton's relativity principle in the inertial reference systems. This principle is formulated as follows: "The equations of motion of the same process for all kinematic reference systems are covariant relative to the certain transformation of these reference systems; moreover this transformation forms a group". The selection of such transformations of reference systems is ambiguous and determined as far as the selection of the corresponding kinematics.

Let $O'x'y'z'$ will be one of the kinematic reference system for the material particle $M$, equation of motion (1) of which in inertial reference system $Oxyz$ is written above. Then, in agreement with the transformation of Galileo,

$$\bar{r} = \bar{u}'t + \bar{r}', \quad t = t', \quad \bar{v} = \bar{u}' + \bar{v}' \quad (3)$$

equation (1) in $O'x'y'z'$ takes the form

$$m\bar{a}' = \bar{F}(t, \bar{u}'t + \bar{r}', u' + v') \quad (4)$$

where $\bar{u}' = const$ - the speed of motion of the kinematic dashed reference system relative to the inertial undashed one.

Let $O''x''y'z''$ be another kinematic reference system for the same material particle $M$. Then, according to the transformation of Galileo,

$$\bar{r} = \bar{u}''t + \bar{r}'' \quad t = t'', \quad \bar{v} = \bar{u}'' + \bar{v}'' \quad (5)$$

equation (1) in $O''x''y''z''$ takes the form

$$m\bar{a}'' = \bar{F}(t, \bar{u}''t + \bar{r}'', u'' + v'') \quad (6)$$

where $\bar{u}'' = const$ - the speed of motion of the reference system $O''x''y'z''$ relative to $Oxyz$.

Comparing (1) with (4) and (6), we see that the equation of motion of material particle (1) in the inertial reference system $Oxyz$ is not covariant relative to the transformation of Galileo. But equation (4), written in one of the kinematic reference system, is covariant relative to Galileo transformation for all kinematic reference systems, moreover this transformation possesses group properties. Let us prove this statement.

The transformation of Galileo

$$r' = \bar{V}t + r'', \quad t' = t'' = t, \quad v' = \bar{V} + v'' \quad (7)$$

where $V$ - the relative speed of reference systems $O'x'y'z'$ and $O''x''y'z''$,

$$\bar{V} = u'' - u' \quad (8)$$

reduces equation (4) to the form

$$m\bar{a}'' = \bar{F}[(t,(u''-V)t + (\bar{V}t + \bar{r}''),(u''-V) + (V + \mathrm{v}'')] \tag{9}$$

and we obtain equation (6), which confirms the group properties of the Galileo transformation.

**Maxwell field equations are Galileo-covariant for all kinematic reference systems**

The Maxwell field equations in some inertial frame $\Sigma$ has the appearance

$$div\,\bar{E} = 0 \tag{10}$$

$$rot\,\bar{E} = -\frac{\partial \bar{B}}{\partial t}, \tag{11}$$

$$div\,\bar{B} = 0, \tag{12}$$

$$rot\,\bar{B} = \frac{1}{c^2}\frac{\partial \bar{E}}{\partial t} \tag{13}$$

According to the dynamic Galileo-Newton's relativity principle, in any other inertial reference system $O^*x^*y^*z^*$ Maxwell field equations have the same form

$$div\,\bar{E}^* = 0 \tag{14}$$

$$rot\,\bar{E}^* = -\frac{\partial \bar{B}^*}{\partial t}, \tag{15}$$

$$div\,\bar{B}^* = 0, \tag{16}$$

$$rot\,\bar{B}^* = \frac{1}{c^2}\frac{\partial \bar{E}^*}{\partial t} \tag{17}$$

In the electrodynamics theory also another problem has arisen: it is necessary to describe the field (10) - (13) in the kinematic unaccelerated reference system $\Sigma'$. At this, $\Sigma'$ is moving relative to IFR $\Sigma$ with velocity $\bar{u}' = const$. To describe in $\Sigma'$ process, taking place in IRS $\Sigma$, let's apply kinematic transformation of Galileo

$$\bar{r} = \bar{r}' + \bar{u}'t, \qquad t = t' \tag{18}$$

to the set of equations (10)-(13).

Let's derive some auxiliary relations. Applying Galileo transformation (18) to the function $f(\bar{r},t)$, we obtain

$$f(\bar{r},t) = f(\bar{r}' + \bar{u}'t',t') = f'(r',t') \tag{19}$$

Further we find

$$\frac{\partial f}{\partial t} = \frac{\partial f'}{\partial t'}\frac{\partial t'}{\partial t} + \frac{\partial f'}{\partial \bar{r}'}\frac{\partial \bar{r}'}{\partial t}, \tag{20}$$

$$\frac{\partial f}{\partial \bar{r}} = \frac{\partial f'}{\partial \bar{r}'}\frac{\partial \bar{r}'}{\partial \bar{r}}. \tag{21}$$

Taking into account, that

$$\frac{\partial t'}{\partial t} = 1,\ \frac{\partial r'}{\partial t} = -\bar{u}',\ \frac{\partial \bar{r}'}{\partial \bar{r}} = \delta_{ij}\ (\ \delta_{ij} = 1, i = j;\ \ \delta_{ij} = 0, i \neq j) \tag{22}$$

from (20) and (21) we get the following relations between the operators in dashed and non-dashed reference systems

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \overline{u}'\overline{\nabla}', \quad \overline{\nabla} = \overline{\nabla}'. \tag{23}$$

With account of (23), Maxwell equations (10)-(13), upon application of Galileo transformation to them, take the following appearance in the reference system $\sum'$

$$div\overline{E}' = 0, \tag{24}$$

$$rot\,\overline{E}' == -\frac{\partial}{\partial t'}\overline{B}' + \left[(\overline{u}'\overline{\nabla}')\overline{B}'\right], \tag{25}$$

$$div\,\overline{B}' = 0, \tag{26}$$

$$rot\,\overline{B}' = \frac{1}{c^2}\frac{\partial}{\partial t'}\overline{E}' - \left[\frac{1}{c^2}(\overline{u}'\overline{\nabla}')\overline{E}'\right]. \tag{27}$$

As expected, the non-covariance Maxwell equations (10)-(13) and (24)-(27) are conditioned by the convective current and convective derivative of field vectors - members in square brackets. But Maxwell equations (24)-(27) are covariant for all kinematic reference systems relative to the transformations of Galileo.

We can conclude, that as in mechanics, so in electrodynamics: a) the dynamic inertial reference systems are equal in the sense that identical physical processes in each of them are flowed and described identically; b) the kinematic reference systems are equal in the sense that the equations of motion of one and the same process in them are covariant relative to their mutual transformations, moreover this transformations possess the group properties.

## Maxwell field equations are Lorentz-covariant for all kinematic reference systems

The first axiom of SRT is not generalization of the experimental Galileo-Newton relativity principle to the electrodynamics processes, as was expected by Einstein, but only the mathematical requirement of covariance of equations of physics relative to the Lorentz transformations.

In works of Lorentz by the formal mathematical computations is showed that Maxwell's equations (10)-(13) in one of the inertial frame, taken as fixed one, retain their form also in another inertial frame. In this case, the coordinates and time in this another inertial frame are combined with the coordinates and time in the first inertial frame by some relationships. But Lorentz did not examine Maxwell's equations in the kinematic reference systems. Einstein paid attention to this fact, and showed that Maxwell's equations preserve the same form at transformation of Lorentz for all kinematic unaccelerated reference systems. And this form is the same, as in inertial reference systems. With this, Einstein showed that the formal mathematical relationships of Lorentz acquire the physical sense of the transformations of space and time in the new kinematics. Lorentz commented the new approach of Einstein as follows : “The special attention should be paid to the remarkable reversibility, which Einstein indicated. Until now the investigations of the phenomena have been carried out only by the observer in the fixed system $A_0$, whereas $A$ was limited to the mobile system $S$ … Einstein focused special attention on this circumstance in his theory, in which he proceeds from the fact that he calls the relativity principle, i.e., the principle, on the basis of which equations, with the aid of which the physical phenomena can be described, do not change their form upon transition from one reference system to another, having uniform rectilinear motion with respect to the initial system” [4]. Such understanding of the relativity principle as the requirement of covariance, is laid into the foundation of the SRT, on what Einstein focused attention repeatedly: “In accordance with the Special relativity theory, the laws of nature must be covariant relative to the Lorentz transformations” [5].

The kinematic relativity principle of Einstein principally differs from the dynamic relativity principle of Galileo-Newton. With this taken into account, it becomes also clear, that the second axiom of the Special relativity theory of Einstein is assertion not of the constancy of the velocity of light in all inertial reference systems, but that about the invariance of this velocity in the kinematic reference systems relative to the transformations of Lorentz. Let us to prove this assertion.

If $\mathrm{v}$ is the phase speed of the wave in inertial frame $Oxyz$ for any unmoved medium, $\mathrm{v}'$ is the velocity of this wave in the kinematic reference system $O'x'y'z'$ and $u'$ is the relative velocity of reference systems $O'x'y'z'$ and $Oxyz$, then in accordance with Lorentz transformations we have

$$\mathrm{v}' = \frac{\mathrm{v} - u'}{1 - \frac{\mathrm{v}u'}{c^2}} \tag{28}$$

If this wave is the light wave, then $\mathrm{v} = c$ and from (28) we obtain

$$\mathrm{v}' = c = inv \tag{29}$$

This invariance of light velocity $\mathrm{v}'$ for all kinematic reference systems is due to the mathematical "deformations" of space and time coordinates in Lorentz transformations.

As a result, after these correct formulations of Einstein's axioms, his Special Relativity Theory becomes the physically substantiated theory, devoid of contradictions, whose mathematical apparatus leads to the same correct results of the electrodynamics of the moving bodies, as the method of the retarding potentials. Always only one point should be taken into consideration, to what reference system the problem is solved in - dynamic or kinematic. This is especially important at interpretation of the obtained results. Ignoring this circumstance leads to the contradictions and paradoxes, being the objects of the critics of Einstein's theory from the moment of its appearance up to our days.


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## The journal "Annalen der Physik"

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As a result, after these correct formulations of Einstein's axioms, his Special Relativity Theory becomes the physically substantiated theory, devoid of contradictions, whose mathematical apparatus leads to the same correct results of the electrodynamics of the moving bodies, as the method of the retarding potentials. Always only one point should be taken into consideration, to what reference system the problem is solved in - dynamic or kinematic. This is especially important at interpretation of the obtained results. Ignoring this circumstance leads to the contradictions and paradoxes, being the objects of the critics of Einstein's theory from the moment of its appearance up to our days.

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Best wishes,
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Dear Professor Potjekhin,

I can confirm that I received your e-mail dated 1 April 2008. However, the present paper is only a slight variation of the previously submitted one, which has been rejected by Professor Kramer. (See also my e-mail dated 17 March 2008.)

I see no reason to revise our earlier decision.

Best regards
Ulrich Eckern

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