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MAXWELL FIELD EQUATIONS ARE GALILEO COVARIANT FOR ALL KINEMATIC REFRENCE SYSTEMS

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Abstract

The concepts of dynamical and kinematical reference systems by Newton are introduced [1], [2]**. The Maxwell field equations in the kinematical non-accelerated (in relation to the inertial) reference systems are derived. It is shown, that Maxwell equations, written in the inertial reference system, are not covariant relative to the transformation of Galileo. But these equations are Galileo-covariant relative for all kinematical reference systems.**

ON THE CLASSIFICATION OF THE REFERENCE SYSTEMS IN PHYSICS

According to Newton all movable reference systems are divided into two classes – dynamic and kinematic [1], [2]. If the considered system of material particles is moving simultaneously with the reference system $\sum$, which, in turn, is moving relative to the sphere of remote stars, then $\sum$ is called a dynamic reference system for the given process. If the considered system of material particles does not participate in the transport motion together with the reference system $\sum'$, then the latter is called kinematic reference system for the given process. It is worth to note the relativity of these concepts: the same reference system for one process can be dynamic, while for the others – the kinematic.

The dynamic reference systems, in turn, are subdivided into two classes - inertial and non-inertial ones. Just in dynamics a concept of inertial reference systems appears. Dynamic reference systems, which move translational, uniformly and rectilinearly concerning the sphere of remote stars, therefore, concerning each other as well, are called the inertial reference systems. In each of such dynamic inertial reference systems, according to the experimentally established dynamic relativity principle of Galileo-Newton, physical laws of not mechanics only, but also of the electrodynamics, are stated and written down equally to within notation of coordinates. Equations of motion of physical processes in dynamic inertial reference systems never include speed of their motion relative to the other reference systems. The dynamic reference systems, which move with acceleration with respect to the inertial dynamic reference systems, are called the non-inertial dynamic reference systems.

The subdivision of kinematical reference systems into non-accelerated and accelerated is conditional (relative) as all of them are equal in rights and any of them can be taken as a stationary one. If the equation of movement of some process relative to one of the reference systems, taken as a stationary, is known, the equation of movement of this process relative to any other kinematical (for the given process) reference systems can be obtained by kinematical transformation of these reference systems, for example, by Lorentz transformation [3] or, as we'll shown bottom, by Galileo transformation. Equation of motion of physical process in kinematical frame of reference always includes speed of their motion relative to the other frame of reference

MAXWELL FIELD EOUATIONS IN DYNAMICAL AND KINEMATIAL REFERENCE SYSTEMS

The Maxwell field equations in some inertial frame $\sum$ has the appearance

$$div\,\overline{E} = 0 \tag{1}$$

$$rot\,\overline{E} = -\frac{\partial \overline{B}}{\partial t}, \tag{2}$$

$$div\,\overline{B} = 0, \tag{3}$$

$$rot\,\overline{B} = \frac{1}{c^2}\frac{\partial \overline{E}}{\partial t} \tag{4}$$

According to the dynamic Galileo-Newton's relativity principle, in any other inertial reference system $\sum *$ Maxwell field equations have the same form

$$div\,\overline{E}* = 0 \tag{5}$$

$$rot\,\overline{E}* = -\frac{\partial \overline{B}*}{\partial t}, \tag{6}$$

$$div\,\overline{B}* = 0, \tag{7}$$

$$rot\,\overline{B}* = \frac{1}{c^2}\frac{\partial \overline{E}*}{\partial t} \tag{8}$$

In the electrodynamics theory also another problem has arisen: it is necessary to describe the field (1) - (4) in the kinematic unaccelerated reference system $\sum'$. At this, $\sum'$ is moving relative to IFR $\sum$ with velocity $\overline{u}' = const$. To describe in $\sum'$ process, taking place in IRS $\sum$, let's apply kinematic transformation of Galileo

$$\overline{r} = \overline{r}' + \overline{u}'t, \qquad t = t' \tag{9}$$

to the set of equations (1)-(4).

Let's derive some auxiliary relations. Applying Galileo transformation (9) to the function $f(\overline{r}, t)$, we obtain

$$f(\overline{r}, t) = f(\overline{r}' + \overline{u}'t', t') = f'(r', t') \tag{10}$$

Further we find

$$\frac{\partial f}{\partial t} = \frac{\partial f'}{\partial t'}\frac{\partial t'}{\partial t} + \frac{\partial f'}{\partial \overline{r}'}\frac{\partial \overline{r}'}{\partial t}, \tag{11}$$

$$\frac{\partial f}{\partial \overline{r}} = \frac{\partial f'}{\partial \overline{r}'}\frac{\partial \overline{r}'}{\partial \overline{r}}. \tag{12}$$

Taking into account, that

$$\frac{\partial t'}{\partial t} = 1, \;\; \frac{\partial r'}{\partial t} = -\overline{u}', \;\; \frac{\partial \overline{r}'}{\partial \overline{r}} = \delta, \; (\delta_{ij} = 1, i = j; \;\; \delta_{ij} = 0, i \neq j) \tag{13}$$

from (11) and (12) we get the following relations between the operators in dashed and non-dashed reference

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \overline{u}'\overline{\nabla}', \quad \overline{\nabla} = \overline{\nabla}'. \tag{14}$$

With account of (14), Maxwell equations (1)-(4), upon application of Galileo transformation to them, take the following appearance in the reference system $\sum'$

$$div\overline{E}' = 0, \tag{15}$$

$$rot\,\overline{E}' == -\frac{\partial}{\partial t'}\overline{B}' + \left[(\overline{u}'\overline{\nabla})\overline{B}'\right], \tag{16}$$

$$div\,\overline{B}' = 0, \tag{17}$$

$$rot\,\overline{B}' = \frac{1}{c^2}\frac{\partial}{\partial t'}\overline{E}' - \left[\frac{1}{c^2}(\overline{u}'\overline{\nabla})\overline{E}'\right]. \tag{18}$$

As expected, the non-covariance Maxwell equations (1)-(4) and (15)-(18) are conditioned by the convective current and convective derivative of field vectors - members in square brackets.

In the kinematic frames of reference there exists a mathematical (theoretical) principle, analogous to the physical (experimental) Galileo-Newton's principle of relativity in the inertial frames of reference. This mathematical principle is formulated as follows: "The equations of motion of the same process for all kinematic frames of reference are covariant relative to the certain transformation of these frames of reference; moreover this transformation forms a group".

Let $O''x''y'z''$ be another kinematic frame of reference. Then, according to the transformation of Galileo,

$$\overline{r} = \overline{u}''t + \overline{r}'' \quad t = t'' \tag{19}$$

and equations (1)- (4) in $O''x''y''z''$ takes the form

$$div\overline{E}'' = 0, \tag{20}$$

$$rot\,\overline{E}'' == -\frac{\partial}{\partial t'}\overline{B}'' + \left[(\overline{u}''\overline{\nabla})\overline{B}''\right], \tag{21}$$

$$div\,\overline{B}' = 0, \tag{22}$$

$$rot\,\overline{B}'' = \frac{1}{c^2}\frac{\partial}{\partial t'}\overline{E}'' - \left[\frac{1}{c^2}(\overline{u}''\overline{\nabla})\overline{E}''\right]. \tag{23}$$

where $\overline{u}'' = const$ - the speed of motion of the frame of reference $O''x''y'z''$ relative to $Oxyz$.

The transformation of Galileo

$$r' = \mathrm{v}t + r'', \quad t' = t'' = t, \tag{24}$$

Where v - the relative speed of frames of reference $O'x'y'z'$ and $O''x''y'z''$,

$$\mathrm{v} = u'' - u' \tag{25}$$

reduces equations (15)- (18) to the equations (20)- (23), that confirms the group properties of the Galileo transformation.

We can conclude, that in electrodynamics, as in mechanics:

a) The dynamic inertial reference systems are equal in the sense that identical physical processes in each of them are flowed and described identically.

b) The kinematic reference systems are equal in the sense that the equations of motion of one and the same process in them are covariant relative to their mutual transformations; moreover these transformations possess the group properties.

References

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